PARAMETER ESTIMATION FROM RADAR TRACKED FLIGHT DATA OF AN ARTILLERY SHELL

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CERTIFICATE

It is certified that the work contained in this thesis entitled, **"Parameter Estimation from Radar Tracked Flight Data of an Artillery Shell"** by I. V. S. Sridhar has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

An attempt is made to estimate aerodynamic parameters using radar tracked coordinates of a free flight artillery shell. The present thesis proposes schemes to estimate average values of aerodynamic parameters and their functional dependence on Mach number using Maximum Likelihood method with limited number of instrumented data. The suitability and applicability of the proposed schemes are evaluated by applying them to 2-dimensional and 3-dimensional flight data.

The proposed schemes were firstly validated using 2-dimensional simulated flight data and as the work progressed, the trajectory model was made 3-dimensional. The results obtained using these schemes show good accuracy of estimated parameters. The Cramer-Rao bounds were low even when moderate noise was included. However limitations of few schemes to deal with limited flight data for parameter estimation purpose is also highlighted.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

C_L, C_{d0}, C_m	= Non-dimensional lift, drag and pitching moment coefficients.
C_N, C_A	= Coefficient of normal and axial force.
$C_{x,} C_y, C_z$	= Coefficient of longitudinal, lateral, vertical force.
d	= Diameter of the artillery shell, mm.
g	= Acceleration due to gravity, m/s^2.
I_x, I_y	= Moment of inertia about x and y axis, kgm^2 .
m	= Mass of the shell, kg.
p	= Roll rate, rad/s.
P	= Ballistic Air Pressure (ambient Air Pressure), mm Hg
q	= Pitch rate, rad/s.
$\bar{q}$	= Dynamic pressure, N/m^2.
r	= Yaw rate, rad/s.
r_e	= Distance from earth center to C.G. of the shell, m.
R	= Radius of earth, m.
s	= Reference area of the artillery shell, m^2.
t	= Time of flight, sec.
T	= Ballistic air temperature (ambient air temperature), ^{o}C.
u, v, w	= Velocity components in x, y and z body axes, m/s.
V	= Air relative speed, m/s.
W_x, W_y, W_z	= Head/tailwind, crosswind and vertical wind components, m/s.
x, y, z	= Spatial coordinates, m.
α_r	= Yaw of repose.
ω	= Rotation vectors.
Ω	= Earth rotation angular velocity, rad/s.

ρ = Density of air, kg/m^3.

θ = Firing Table Elevation, Pitch attitude, deg.

ϕ = Roll attitude, deg.

ψ = Yaw angle, deg.

α = Angle of attack, deg.

β = Angle of sideslip, rad.

Superscripts

$\bullet$ = Derivative with respect to time.

$\rightarrow$ = Vector quantity.

Subscripts

o = Initial conditions.

x, y, z = Components along x, y and z direction.

$wind$ = Wind axes.

Stability and control derivatives

$$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha}, \qquad C_{Lq} = \frac{\partial C_L}{\partial (qd/2V)}, \qquad C_{L\delta} = \frac{\partial C_L}{\partial \delta}.$$

$$C_{m\alpha} = \frac{\partial C_m}{\partial \alpha}, \qquad C_{mq} = \frac{\partial C_m}{\partial (qd/2V)}.$$

$$C_{l\delta} = \frac{\partial C_l}{\partial \delta}, \qquad C_{lp} = \frac{\partial C_l}{\partial (pd/2V)}.$$

$$C_{n\beta} = \frac{\partial C_n}{\partial \beta}, \qquad C_{nr} = \frac{\partial C_n}{\partial (rd/2V)}.$$

$$C_{y\beta} = \frac{\partial C_y}{\partial \beta}, \qquad C_{yr} = \frac{\partial C_y}{\partial (rd/2V)}.$$

CHAPTER 1
INTRODUCTION

Artillery forms an important wing of the army to provide firepower during war as well as during cross-border skirmishes with the enemy. Artillery generally falls into three basic categories: guns, howitzers and mortars. The principle difference between them being the trajectory of the round fired. A gun has a high muzzle velocity and a very flat trajectory. Normally a gun is used in a direct fire mode where the target can be seen and penetration is desirable. Howitzers have a somewhat lower muzzle velocity and arc their shells onto a target. They are used in both a direct fire and indirect fire mode. This is especially useful when an enemy is concealed behind a prepared position or the artillery men desire to have a shell explode over an enemy's head. The airburst does less damage to hardened targets, but causes many more human casualties due to shell fragmentation covering a large area. Mortars have a very pronounced arc of flight. They have a relatively low muzzle velocity and are unsuitable for direct fire. Their principle value comes from being able to lob shells behind an obstacle, such as a fortification or a hill. They are not very accurate and dependent upon the amount of propelling powder to determine the point of impact.

The effectiveness of artillery is largely judged by the accuracy in hitting the targets. The accuracy and reliability is influenced by the design criteria used in designing the shell. Artillery shells are a class of projectiles around which much of the aeroballistic theory was originally developed, and they continue to form a significant part of the aeroballistician's interest.

Study of the motion of projectiles through an external medium is known by the name of external ballistics. If the external medium is the earth's atmosphere, then external ballistics become synonymous with aeroballistics.

The conventional approach hitherto for understanding the in-flight behaviour of projectiles was to develop mathematical models that could predict all elements of the trajectory from launch to target. To this purpose, it becomes essential that all forces, moments affecting the flight of the projectile be accounted for in a well-defined mathematical form[1]. Beginning with the most simple but relatively inaccurate mathematical model, the in-vacuo trajectory model, more and more sophisticated models of increasing accuracy such as the point mass model (PMM), the modified point mass model (MPMM) and the six-degrees-of-freedom (SDF) model have been developed[1]. These models require aerodynamic coefficients as input (e.g., drag coefficient C_d, damping in roll derivative C_{l_p}, lift curve slope $C_{L\alpha}$, etc.). Accurate values of these parameters are required in mathematical model for description of artillery shell dynamics.

There are three distinct approaches for estimating aerodynamic parameters:

- Theoretical methods.
- Wind tunnel testing.
- Flight testing.

At the preliminary design stage of any system, theoretical methods[2, 3, 4] are useful inspite of their limited accuracy. The wind tunnel testing improves the accuracy of estimation but it is a time consuming and expensive way of estimating the aerodynamic parameters. Precise simulation of control surfaces, power effects and flight conditions is difficult. The model tested in the wind tunnel is generally slightly different from actual

flight due to last minute configuration changes. Other reasons for discrepancies between flight and wind tunnel results are Reynolds number discrepancies and interferences due to support system. It is therefore desirable that the wind tunnel estimates be corroborated with the estimates from actual flight test data.

The most commonly used methods for parameter estimation from flight data are broadly classified into the following categories:

- Equation error methods.
- Output error methods.

The principle of Least squares is used in the equation error method wherein the error gets minimized with respect to the unknown parameters in each of the equations. Its advantages include computational simplicity, non-iterative nature and applicability to both linear and non-linear models. The disadvantage is that the method cannot be directly applied if all the states are not measured accurately, and produces poor results if the measurements are noisy. Therefore in order to get accurate results, considerable effort is required for data reconstruction and smoothing. Sometimes these data processing tasks are more complicated than the task of parameter estimation itself. However these methods are useful as startup for more advanced estimators.

In the output error method, the error between the measured and the model response is minimized. The method assumes that there exist no modeling errors. This method processes the measurement noise while assuming the model representation of the given system to be exact. A comprehensive survey of these methods is reported by Maine and

Iliff[5]. The methods like analog matching[6] , Newton Raphson method[7] , modified Newton Raphson method[7] etc. fall in this category.

Advanced methods are capable of estimating parameters in the presence of measurement and/or process noise. Maximum Likelihood (ML) estimates are those for which the observed value would be the most likely to occur; "most likely" is defined as maximization of Likelihood function of the observed responses, given the parameters. The main advantage of this method is that parameter estimates are asymptotically unbiased, consistent and efficient provided the model assumed is correct.

Estimation activities in aerospace science started in late fifties. Though plenty of literatures on parameter estimation exist for aircraft very few exist for artillery shells. It is worth mentioning that parameter estimation technique is a blend of both science and art. Parameter estimation of an aircraft with many measurements like, acceleration (both linear and angular), angular orientation, speed, angle of attack etc. has been discussed in detail in Ref.8. But from cost effectiveness point of view (and also because of non-availability of space for instrumentation), it may not be feasible to use many sensors for artillery shells going through many development flight trials.

As an alternative approach, limited flight tests are conducted with scale down model in Aero-ballistic range facility[9]. The test facility is an enclosed, atmospheric, instrumented concrete structure used to examine the exterior ballistics of various free flight configurations. The facility contains gunroom, control room, model measurement room, blast chamber and instrumented range. The range may have large number of locations available as instrumentation sites. Each station is used to house fully instrumented orthogonal shadowgraph stations. Using this instrumentation, spatial

positions, speed and angular orientations etc. are estimated. These are then used for parameter estimation.

Winchebach et al [9] conducted free flight aeroballistic tests to obtain subsonic and supersonic aerodynamic parameters of a wrap around fin configuration. The aerodynamic parameters presented were extracted from the position–attitude time histories of the experimentally measured trajectories using non-linear numerical interpolation data reduction routines.

Defense Research Establishment, Quebec, Canada is currently engaged in research on high l/d energy penetrators for 70mm air launched rocket application. One of the objectives of the program is to enlarge the free-flight aerodynamic database of high l/d finned projectiles[10]. Dupis investigated six model configurations (scale down) during the test program[10]. Standard SDF routine with ML method was used to match the theoretical trajectory to the experimentally measured trajectory. In that analysis, as a first step, the basic dynamic range data (time, position, angles), physical properties and atmosphere conditions were assembled and then SDF (with ML) were performed to estimate the aerodynamic parameters[10]. The results of this analysis showed that once time histories of position, angles are recorded it is possible to estimate aerodynamic parameters using free flight test data.

It is worth mentioning that aerodynamic parameters are strong functions of Mach number. In order to capture this dependence on Mach number, projectiles are fired at different initial Mach number and the basic data corresponding to that Mach number are recorded and analyzed for parameter estimation[9,10]. Though this approach looks attractive (if such facility is available), however it has following two distinct limitations:

1) It may not be possible to simulate actual flight path that the full-scale model would be traversing.
2) It requires large number of firings to capture Mach number dependence of aerodynamic parameters.

In view of the above, there have been conscious efforts towards generation of flight data of free flight projectile by simultaneous measurements with pulse radar or optical system for position data and continuous wave Doppler radar for velocity measurements. In order to improve the overall accuracy several systems of each type have often been used in parallel[11]. However, this complicated and very expensive set up is not required today. From a single continuum wave Monopulse Azimuth and Elevation Doppler Tracking Radar it is possible to extract highly accurate velocity and position data in three dimensions without any assumptions of ballistic behaviour[11].

Defense Research Organization of India, is currently engaged in design and development of new artillery shell, rockets etc. There is a routine demand for post flight analysis of flight data. In particular, there is strong demand for estimation of aerodynamic parameters from flight data to validate the design as well as to prepare database for fire control system (FCS) to enhance artillery effectiveness. Accurate values of aerodynamic parameters are of paramount importance in building effective FCS.

Due to non-availability of Aero-ballistic range facility in India, lots of efforts are being directed towards acquiring flight data through Radar tracking. At present, the radar system available can only measure spatial distance (x, y, z) accurately. The available radar does not have facility to measure velocity, especially for long range artillery shell.

Thus, the motivation for this research is inspired by this challenge of estimating aerodynamic parameters using radar tracked spatial coordinates of artillery shell.

Free flight projectiles experience aerodynamic forces and moments as it traverse from launch point to the target end. Bryan[12] introduced the concept of flight vehicle aerodynamic modeling, which provided relationship between the forces along the three Cartesian axes and the three moments about these axes as a function of linear translational motion variables and rotational rates. The most commonly employed aerodynamic models are typically formulated as truncated Taylor-series expansion of the form,

$$C_i = \sum_i \frac{\partial C_i}{\partial X} X; \qquad i = x, y, z, l, m, n.$$

where X are the vehicle motion variables (angle of attack, angle of sideslip, angular rates etc.).

Conventionally, application of ML method will require measurements of motion variables for the estimation of the aerodynamic derivates (parameters, $\frac{\partial C_i}{\partial X}$). However, for the problem chosen in this thesis, we do not have the information of these motion variables. The proposed method uses only the information about the spatial location of the shell to estimate aerodynamic parameters.

Artillery shells are fired at high muzzle velocity (2-3 Mach) and are expected to reach maximum range at an angle of elevation of around 45 degrees. During this motion it encompasses high supersonic to low subsonic Mach numbers. The aerodynamic parameters are strong function of Mach number.

In order to develop accurate mathematical model, it is necessary to capture the Mach number dependence of these parameters. In order to capture their dependence, several schemes have been proposed to estimate aerodynamic parameters from tracked data (x, y, z) of an artillery shell. Through the proposed schemes, attempt has been made to estimate equivalent (constant) values of aerodynamic parameters that are expected to account for the dependence of parameters on Mach number. Further schemes have been proposed to capture the Mach number dependence of these parameters by modeling the functional form of their variation in estimation algorithm.

Due to non-availability of the real flight data, all the schemes as stated above were tested on simulated flight data. To start with, simulated data was generated using PMM. This is a simplified model and only uses drag coefficient (C_d) for trajectory computation. The proposed schemes were applied to estimate equivalent as well as Mach number dependent of C_d successfully. Next the simulated data was updated by incorporating three more aerodynamic parameters by generating trajectory data using MPM model. Proposed schemes were applied to estimate the aerodynamic parameters. Finally, the simulated flight data was generated using SDF model, and proposed schemes were applied to investigate its capability to estimate large number of aerodynamic parameters used in simulation. The difficulties encountered are also presented as the work concluded with future suggestions.

The work carried out in this thesis is presented in 4 chapters. Chapter 1 consists of brief introduction about parameter estimation and Chapter 2 describes various schemes proposed and generation of simulated flight data for estimation. Chapter 3 consists of results and discussions followed by conclusions and future suggestions in Chapter 4.

CHAPTER 2

PROPOSED SCHEMES AND GENERATION OF FLIGHT DATA

2.1 General

This chapter describes the various schemes proposed and different types of simulated flight data generated for validating the proposed schemes. Various mathematical models consisting of the equations of motion that had been used for the generation of flight data are being outlined.

2.2 Proposed schemes

The following schemes have been proposed for estimation of aerodynamic parameters from flight data of an artillery shell using ML method:

Scheme 1: In this scheme motivation was to estimate equivalent parameters, which can be assumed to be an approximation for the variation of parameters with Mach number. In order to validate this scheme flight data were generated, where the variation of aerodynamic parameters with Mach number were used in the simulation model. However, the Mach number dependence was not modeled in estimation algorithm, and thus estimated parameters (equivalent parameters) were expected to include the effects of Mach number variation in some approximate manner.

Scheme 2: In this scheme motivation was to estimate the functional dependence of parameters on Mach number. In order to validate this scheme flight data were generated by incorporating variation of aerodynamic parameters with Mach number (as supplied by the manufacturer) in simulation model. However, in the estimation algorithm

the Mach number dependence of aerodynamic parameters were approximated by expressing aerodynamic parameters as a polynomial function of Mach number. The constants of this polynomial series were estimated applying ML method.

Scheme 3: In this scheme motivation was same as that of scheme 2 but different methodology was used. The whole trajectory was split into different sets of points and average values of the parameters pertaining to each set of points were estimated. The estimated parameters correspond to average Mach number of the different set of points used for estimation.

Scheme 4: Finally, in this scheme, the flight data was generated at a particular elevation corresponding to different charges (muzzle velocities), and small initial part of the trajectory was used to estimate the average values of aerodynamic parameters. These estimated parameters correspond to the Mach number corresponding to different muzzle velocities.

As the simulation was made exhaustive by incorporating large number of parameters, few of the schemes faced difficulties to estimate all the parameters. Flight data were generated using all the three trajectory models as explained in next section. The aerodynamic and geometric characteristics of the artlillery shell used for generating the simulated flight data are listed in Table 2.1 and Table 2.2 respectively.

Table 2.1 Aerodynamic characteristics of the artillery shell.

SL. No.	Mach Number	C_d	$C_{m\alpha}$	$C_{L\alpha}$	C_{lp}
1.	0	0.13	3.2	1.3	-0.037
2.	0.85	0.13	3.2	1.3	-0.031
3.	0.9	0.13	3.25	1.2	-0.03
4.	0.92	0.14	3.3	1.1	-0.03
5.	0.96	0.203	3.45	0.95	-0.03
6.	0.98	0.269	3.5	0.95	-0.03
7.	1	0.329	3.5	1	-0.0297
8.	1.02	0.351	3.4	1.1	-0.0295
9.	1.05	0.367	3.25	1.55	-0.029
10.	1.1	0.366	3.15	1.85	-0.029
11.	1.15	0.353	3.15	1.95	-0.0285
12.	1:2	0.343	3.14	2	-0.0282
13.	1.3	0.326	3.13	2.07	-0.028
14.	1.4	0.314	3.12	2.15	-0.027
15.	1.6	0.293	2.75	2.2	-0.026
16.	1.8	0.273	2.5	2.3	-0.025
17.	2	0.257	2.48	2.32	-0.024
18.	2.2	0.245	2.47	2.35	-0.023
19.	2.4	0.236	2.45	2.35	-0.022
20.	2.6	0.229	2.4	2.35	-0.0215

Table 2.2 Geometric characteristics of the artillery shell.

SL. No.	Geometric characteristics
1.	Total length : 825 mm.
2.	CG location : 533 mm from nose tip.
3.	Diameter : 155 mm.
4.	Mass : 42.6 kg.
5.	Moment of Inertia Axial, I_{xx} : 0.146 kg-m^2 . Traverse, I_{yy} :1.709 kg-m^2 .

2.3 Trajectory models used for simulating flight data

In general, the trajectory followed by center of mass of artillery shell in space is 3-Dimensional (x, y, z). Theoretical modeling of such trajectory demands aerodynamic modeling to be exhaustive. It requires usage of large number of aerodynamic parameters. Hence estimation of all parameters at one time using proposed schemes might be involved. It was thus decided firstly to validate/investigate the proposed schemes using theoretically simulated 2-Dimensional flight data. As the work progressed, the trajectory model was made 3-Dimensional and all the schemes were tried. Accordingly the work started using simulated data through Point Mass Model (PMM) and then extended to Modified Point Mass Model (MPMM) and Six Degree of Freedom model (SDF) [1].

2.3.1 Point Mass Model (PMM)

In this model, it is assumed that the only aerodynamic force acting on the projectile is drag. It provides fairly accurate estimates of range for adequately stable projectiles and can also be used to estimate the first order effects of wind. The equations of motion for point mass model are given as below:

$$\frac{d^2x}{dt^2} = -\frac{\pi \rho d^2 C_d}{8m} V \left(\frac{dx}{dt} - W_x \right), \tag{2.1}$$

$$\frac{d^2y}{dt^2} = -g - \frac{\pi \rho d^2 C_d}{8m} V \left(\frac{dy}{dt} - W_y \right), \tag{2.2}$$

$$\frac{d^2z}{dt^2} = -\frac{\pi \rho d^2 C_d}{8m} V \left(\frac{dz}{dt} - W_z \right). \tag{2.3}$$

where x denotes range, y denotes height, z denotes drift and W_x, W_y, W_z respectively, the x, y and z components of the wind velocity W. This model, however, does not account for the effect of spin of the shell and thus fails to predict the drift due to yaw of repose moment. Also, it neglects the lift forces acting on the shell.

2.3.2 Modified Point Mass Model (MPMM)

The modified point mass model is also known as four degree-of-freedom model (three spatial degrees-of-freedom plus axial spin). Its basis is a conventional point mass model, in addition, the instantaneous equilibrium yaw is calculated at each time step along the trajectory so as to provide estimates of yaw, drag, and drift resulting from the yaw of repose. The equations of motion for modified point mass model are given as below:

$$\frac{d\bar{u}}{dt} = -\frac{\pi \rho d^2}{8m}\left(C_{d0} + C_{d\alpha^2}\, \alpha_r^2 \right) v \vec{V} \qquad \text{(Drag)}$$

$$+ \frac{\pi \rho d^2}{8m} C_{L\alpha}\, v^2\, \alpha_r \qquad \text{(Lift)}$$

$$- g_0 \frac{R^2}{r_e^3} \bar{r} + 2\left(\bar{\omega} \times \bar{u} \right) \qquad \text{(Gravity and Coriolis)} \qquad (2.4)$$

$$\frac{dp}{dt} = \frac{\pi \rho d^4}{16 I_x} p v C_{lp} \qquad \text{(Spin damping)} \qquad (2.5)$$

$$\alpha_r = \frac{-8 p I_x}{\pi \rho d^3 C_{m\alpha}} \frac{\left[\vec{V} \times \left(d\bar{u}/dt \right) \right]}{v^4} \qquad \text{(Yaw of repose)} \qquad (2.6)$$

The quantities in the above equations are defined as follows:

$$\vec{u} = \frac{d\vec{x}}{dt}, \qquad \vec{V} = \vec{u} - \vec{W}, \qquad \vec{r} = \vec{x} - \vec{R}$$

$\vec{R} = (0, -R, 0)$, [$\vec{W}$ Denotes the wind vector]

$$\omega = \left(-\Omega\ \cos[\text{latitude}]\ \cos[\text{azimuth}], -\Omega\ \sin[\text{latitude}], \Omega\ \cos[\text{latitude}]\ \sin[\text{azimuth}]\ \right)$$

where, Ω = 7.29×10^{-5} rad / s. (rotation of the earth),

R = 6370320 m (radius of the earth),

g_0 = 9.80665 [1 - 0.0026373 cos (2×latitude) + 0.0000059 [cos (2× latitude)]^2],

the axis system is as for the point mass model with $\vec{x}$ being along the line of fire.

As may be seen from the above equations, the aerodynamic coefficient input required is extensive and accuracy of these aerodynamic coefficients is crucial for reliable estimates of range. However, for most artillery shells, the reliability of estimated values of aerodynamic coefficients is not high enough to inspire confidence in resulting range estimates from this model. In spite of this limitation, this model has the capability of providing reasonable accounts of end point data. It can estimate wind corrections and can accept non-linear aerodynamic inputs if required.

2.3.3 Six Degree of Freedom model (SDF)

As a final step, the most sophisticated trajectory model developed was the six-degree-of-freedom model having the spatial degrees of freedom, yaw degree of freedom in two planes and the spin. In this model there are no assumptions concerning linearised aerodynamics or projectile symmetry. However, the indeterminability of many of the initial conditions and aerodynamic coefficients which are required as input frequently

results in the model not giving significantly better end results than the modified point mass model. Thus its usage might not be justified for routine fire control work. It is nevertheless a powerful tool for the ammunition designer. The equations of motion for six-degree-of-freedom model are given as below:

$$\dot{u} = (\overline{q}s/m)C_x - qw + rv - g\sin\theta + (Th/m) \tag{2.7}$$

$$\dot{v} = (\overline{q}s/m)C_y - ru + pw + g\sin\phi\cos\theta \tag{2.8}$$

$$\dot{w} = (\overline{q}s/m)C_z - pv + qu + g\cos\phi\cos\theta \tag{2.9}$$

$$\dot{p} = \left\{\overline{q}sdC_l + qr\left(I_y - I_z\right)\right\}/I_x \tag{2.10}$$

$$\dot{q} = \left\{\overline{q}sdC_m + rp\left(I_z - I_x\right)\right\}/I_y \tag{2.11}$$

$$\dot{r} = \left\{\overline{q}sdC_n + pq\left(I_x - I_y\right)\right\}/I_z \tag{2.12}$$

$$\dot{\phi} = p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \tag{2.13}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{2.14}$$

$$\dot{\psi} = r\cos\phi\sec\theta + q\sin\phi\sec\theta \tag{2.15}$$

To derive the spatial position equations, above equations were used to transform the body-axis velocity (u, v, w) into earth fixed axis. The equations are:

$$\begin{aligned}\dot{X} = {} & u\cos\psi\cos\theta + v\left(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi\right) \\ & + w\left(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi\right) + W_x\end{aligned} \tag{2.16}$$

$$\dot{Y} = u\sin\psi\cos\theta + v\left(\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi\right) + w\left(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi\right) + W_y \tag{2.17}$$

$$\dot{Z} = u\sin\theta - v\cos\theta\sin\phi - w\cos\theta\cos\phi + W_z \tag{2.18}$$

Aerodynamic model used in this analysis is as given below:

$$Cx = -C_{Dwind}\cos\alpha\cos\beta + C_L\sin\alpha - C_{ywind}\cos\alpha\sin\beta$$

$$Cy = C_{ywind}\cos\beta - C_{Dwind}\sin\beta$$

$$Cz = -C_L\cos\alpha - C_{Dwind}\sin\alpha\cos\beta - C_{ywind}\sin\alpha\sin\beta$$

where,

$$C_L = C_{L0} + C_{L\alpha} + C_{Lq}\left(qd/2V\right)$$

$$C_{ywind} = C_{y0} + C_{y\beta}\,\beta + C_{yr}\left(rd/2V\right)$$

$$C_{Dwind} = C_d$$

$$C_l = C_{lp}\left(pd/2V\right)$$

$$C_m = C_{m0} + C_{m\alpha}\,\alpha + C_{mq}\left(qd/2V\right)$$

$$C_n = C_{n0} + C_{n\beta}\,\beta + C_{nr}\left(rd/2V\right)$$

Wind model used in this analysis is as given below:

$$u' = u - W_x\cos\theta\cos\psi - W_y\cos\theta\sin\psi + W_z\sin\theta$$

$$v' = v - W_x\left(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi\right) - W_y\left(\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi\right) - W_z\cos\theta\sin\phi$$

$$w' = w - W_x\left(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi\right)$$
$$-W_y\left(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi\right)$$
$$-W_z\cos\theta\cos\phi$$

thus, $\alpha = \tan^{-1}\left(\frac{w'}{u'}\right)$ and $\beta = \tan^{-1}\left(\frac{v'}{u'}\right)$

where W_x, W_y, W_z are the wind components blowing toward the x, y, and z directions.

2.4 Generation of simulated flight data

The capability of the proposed schemes to estimate parameters from spatial coordinates of the artillery shell is investigated using the following sets of flight data generated through the above models:

S1: In this case the flight data (t, x, y) were generated through PMM for different elevations using C_d varying with Mach number as listed in Table 2.1. Flight data thus obtained were used to estimate equivalent value of C_d and its dependence on Mach number respectively.

S2: In this case the flight data (t, x, y) were generated using C_d varying with Mach number for elevations 10, 20 so on up to 60 Deg. respectively and only end points (where the projectile strikes the ground) had been recorded for estimation purpose. In most of the field trails, radar tracked data may not be easily available. Conventionally only the end point data are measured and recorded. Hence to investigate the suitability of

the proposed schemes, these were applied on end point data to estimate an equivalent value of C_d.

S3: In order to check robustness of the proposed schemes while dealing noisy data, flight data were corrupted with known noise. Thus, in this case simulated pseudo random noise of varying intensity was added to the flight data that had been generated in S1 which intern was used to estimate the functional dependence of Mach number on C_d. A measure of confidence level was estimated through the estimation of Cramer-Rao bound.

After getting some initial success, the simulation model to generate flight data was progressively made exhaustive by incorporating more number of aerodynamic parameters. 3-Dimensional flight data of an artillery shell were generated using MPMM and SDF models.

S4: In this case the flight data (t, x, y, z) were generated through MPMM using constant value of parameters C_d, $C_{L\alpha}$, $C_{m\alpha}$ and C_{lp}. In this model number of parameters to be estimated are four. These data were used to check suitability of proposed scheme in handling more then one parameter for estimation.

S5: In this case the flight data (t, x, y, z) were generated through MPMM by modeling the variation of aerodynamic parameters (C_d, $C_{L\alpha}$, $C_{m\alpha}$ and C_{lp}) with Mach number. Flight data thus obtained were used to estimate equivalent values of parameters and Mach number dependence of all the parameters.

S6: In this case the flight data (t, x, y, z) were generated through SDF model using constant value of parameters C_d, $C_{L\alpha}$, $C_{m\alpha}$ and C_{lp}. In this model number of parameters to be estimated are still more. So, in order to check the applicability of the proposed schemes, these data were used to test whether the proposed schemes can estimate all these parameters at one time.

S7: In this case the flight data (t, x, y, z) were generated through SDF model by modeling the variation of aerodynamic parameters (C_d, $C_{L\alpha}$, $C_{m\alpha}$ and C_{lp}) with Mach number. Flight data thus obtained were used to estimate equivalent values of parameters and Mach number dependence of all the parameters.

CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 General

In this chapter results of the estimated parameters will be presented for artillery shell utilizing the schemes proposed for flight data in chapter 2. Before presenting the detailed results of estimated parameters and discussing their accuracy, a brief discussion is given below about the initial values, number of data points, sampling rate and intensity of measurement noise.

Different combinations of initial guess values of the parameters were used to start the iterative algorithm of Maximum Likelihood (ML) method. In general, results were found not much affected by the choice of the initial values except for a few cases. It is therefore decided to present results for typical sets of initial values respectively.

The optimal number of data points chosen varies from case to case as will be explained accordingly. The sampling rate had been fixed at t = 0.01 sec for almost all the cases. The simulated flight data, as explained in chapter 2, in terms of spatial coordinates (x, y, z) had been generated and used for estimation.

A study was also carried out to see the effect of measurement noise on the estimation of parameters. Accordingly, simulated pseudo random noise of varying intensity was added to the flight data. The noise was simulated by generating successive uncorrelated pseudo random numbers having normal distribution with zero mean and assigned standard deviation. The intensity of the noise differs from case to case and results are presented for the same. Cramer-Rao bounds had been calculated for both no noise and noise cases and they seem to be well within the limits. This shows high confidence level of the estimated parameters.

3.2 Estimation of equivalent parameters using scheme 1

In this scheme motivation, as discussed earlier, was to estimate equivalent parameters which can be assumed to be an approximation for the variation of parameters with Mach number. This scheme was applied to various simulated flight data generated in order to estimate equivalent parameters as discussed below:

3.2.1 Estimation of equivalent C_d

In order to test the code, flight data were generated using PMM with a constant value of C_d. Using this simulated flight data the value of aerodynamic parameter C_d was estimated using ML method. The estimated value of C_d exactly matched with that of the true value. Table 3.1 presents this result.

Table 3.1 Comparison of estimated and true values of C_d; scheme 1.

SL. No.	Parameter	True Value	Initial Guess Value	Estimated Value
1.	C_d	0.30	0.50 Or 0.10	0.30 (0)*

*Cramer-Rao bound.

After validating the code by above approach, equivalent value of C_d was estimated using the simulated flight data S1. For this purpose, flight data were generated for a set of different elevations and an average value of C_d, pertaining to each elevation, was estimated using scheme 1 as shown in Table 3.2 respectively.

Table 3.2 Equivalent value of C_d estimated; scheme 1, flight data S1.

SL. No.	Elevation (θ), degrees	Equivalent C_d
1.	10	0.267803 (0.0000026)*
2.	15	0.274797 (0.0000015)
3.	20	0.276251 (0.0000010)
4.	25	0.275646 (0.00000077)
5.	30	0.274519 (0.00000061)
6.	35	0.273226 (0.00000050)
7.	40	0.271675 (0.00000042)
8.	45	0.270041 (0.00000037)
9.	50	0.268703 (0.00000033)
10.	55	0.267809 (0.0000003)
11.	60	0.267432 (0.00000028)
12.	65	0.267479 (0.00000026)
13.	70	0.267682 (0.00000026)

*Cramer-Rao bound

Average equivalent value of C_d i.e. 0.271005 was obtained by taking average of individual estimated C_d values. Using this equivalent C_d value, estimated flight data (t, x, y) were compared with that of measured data for different elevations as shown in Fig. 3.1. The error in both range and height between measured and estimated responses are plotted with range as shown in Fig. 3.2. Table 3.3 compares the range between measured and estimated responses for different elevations. Comparing measured and

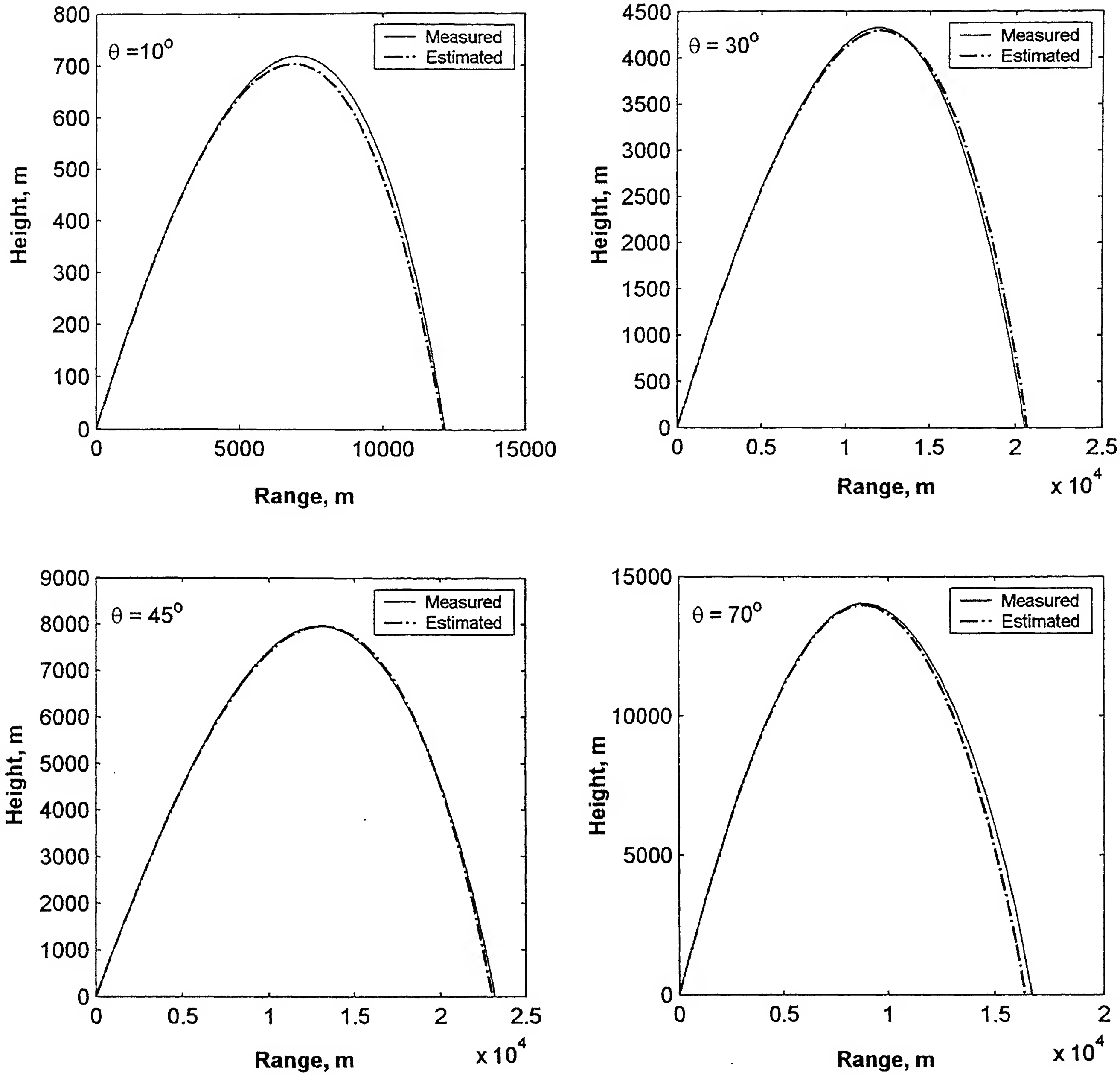

Fig 3.1 Comparison of measured and estimated trajectories obtained using equivalent C_d ; scheme 1, flight data S1.

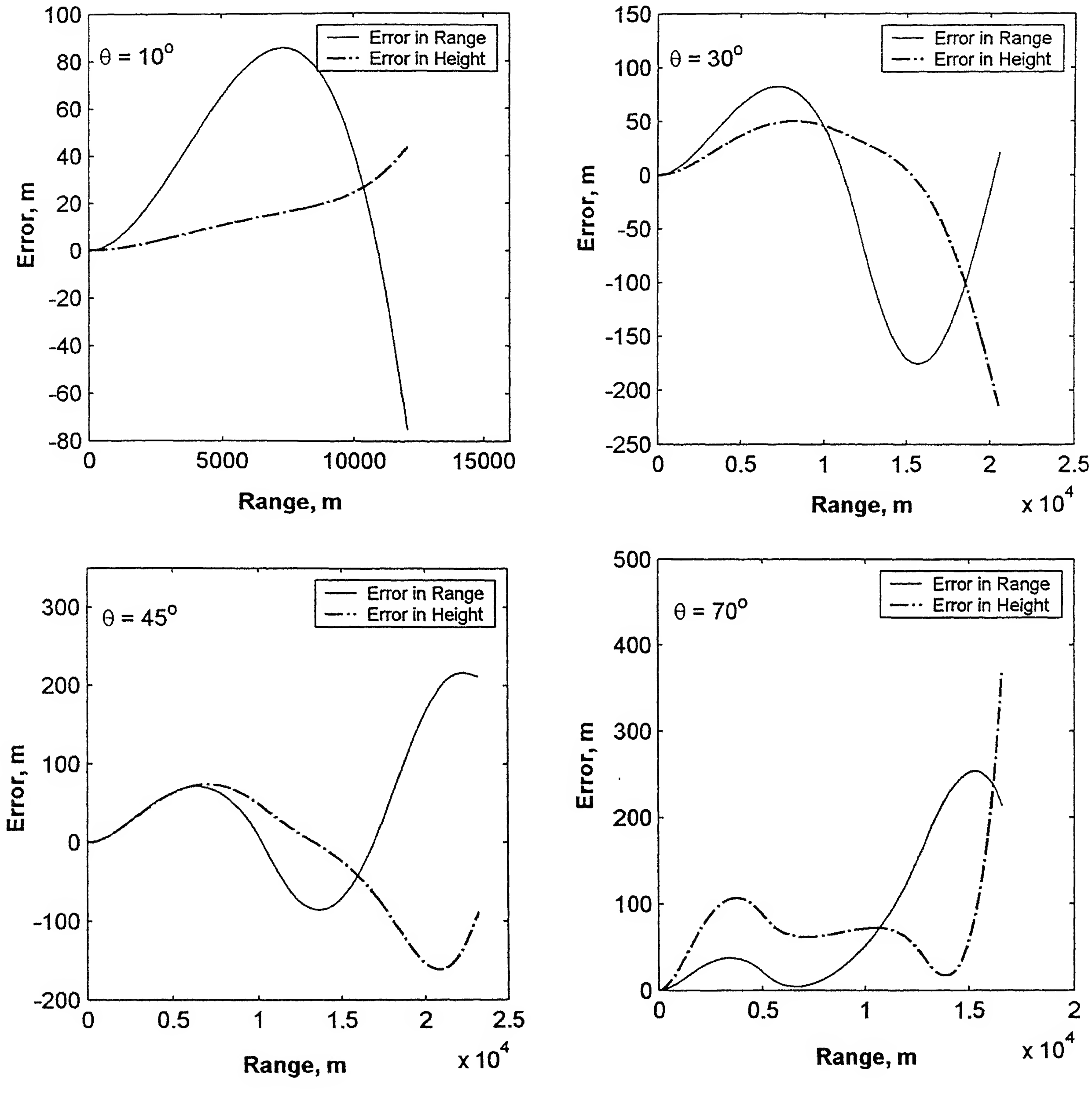

Fig 3.2 Comparison of error in range and height between measured and estimated trajectories obtained using equivalent C_d ; scheme 1, flight data S1.

estimated range, it can be observed that the maximum errors in range and height are well within one percent of the maximum range/height . Further low values of Cramer-Rao bound show higher level of confidence in estimation.

Table 3.3 Comparison of measured and estimated range using equivalent C_d ; Scheme 1, flight data S1.

SL. No.	Elevation (θ), degrees	Measured Range, m	Estimated Range, m
1.	10	12193.078	12131.827
2.	32	21100.430	21229.985
3.	52	23088.499	22773.323
4.	70	16701.777	16403.438

3.2.2 Estimation of equivalent C_d using only end point data

In this case, the flight data were generated as stated in S2. Scheme 1 was applied to estimate equivalent value of C_d using this data. The estimated value of C_d was found to be 0.269074. Using this equivalent C_d value, estimated flight data (t, x, y) were compared with that of measured data for different elevations as shown in Fig. 3.3. The error in both range and height between measured and estimated responses is also plotted with range as shown in Fig. 3.4. Table 3.4 compares the range between measured and estimated responses for different elevations. Notwithstanding the

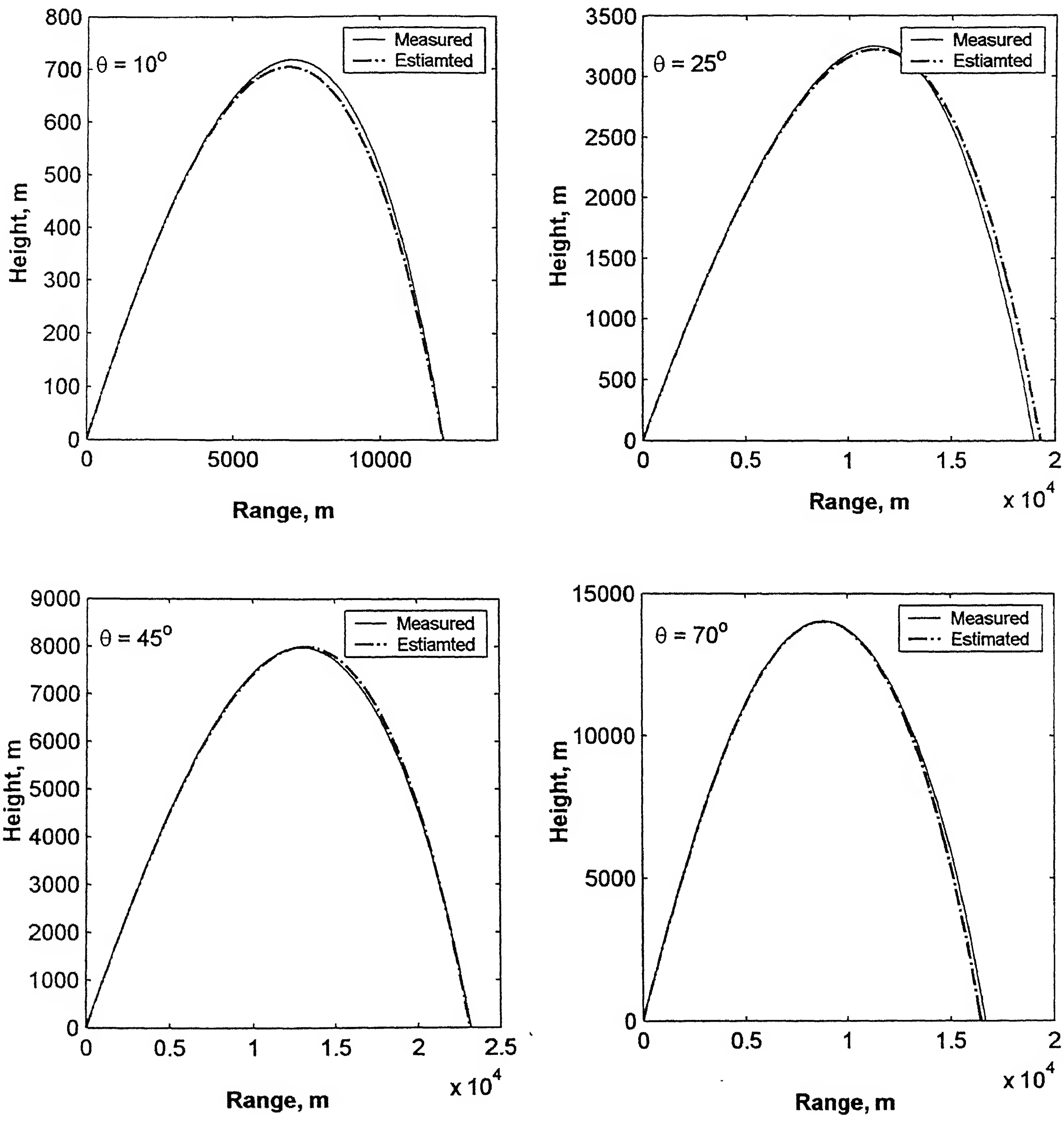

Fig 3.3 Comparison of measured and estimated trajectories using equivalent C_d; scheme 1, flight data S2.

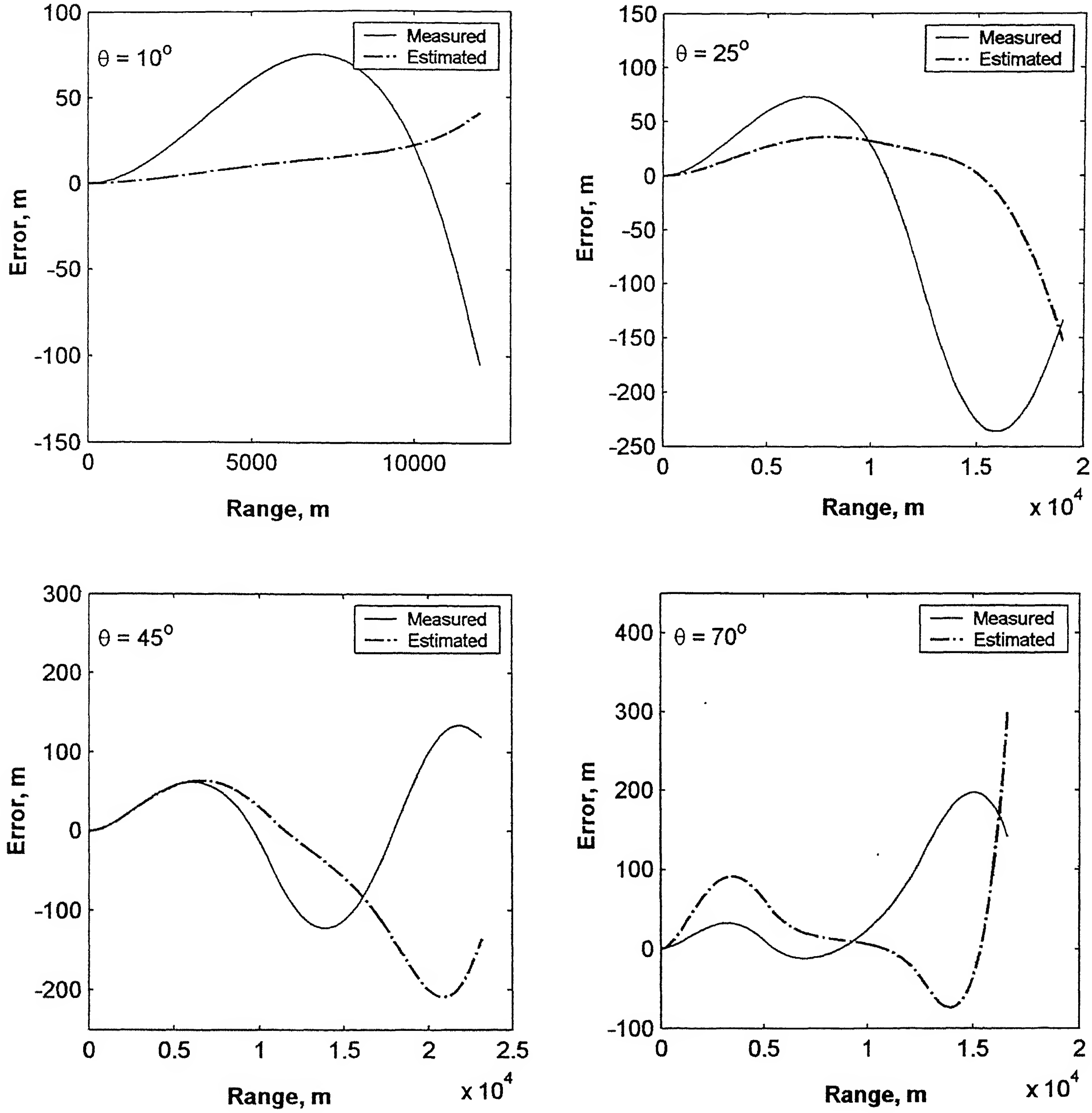

Fig 3.4 Comparison of error in range and height between measured and estimated trajectories using equivalent C_d ; scheme 1, flight data S2.

difference in range, this scheme could be very useful in getting some average value of C_d for a particular type of shell through end point data.

Table 3.4 Comparison of measured and estimated range using equivalent C_d; scheme 1, flight data S2.

SL. No.	Elevation (θ), degrees	Measured Range, m	Estimated Range, m
1.	10	12193.078	12170.877
2.	25	19038.396	19329.929
3.	45	23219.449	23169.223
4.	70	16701.777	16493.113

3.2.3 Estimation of equivalent values of C_d, $C_{L\alpha}$, $C_{m\alpha}$ and C_{lp} from S5

In order to test the code flight data were generated as stated in S4. ML method was applied to estimate these parameters. It was found that the estimation algorithm failed to estimate $C_{L\alpha}$ and $C_{m\alpha}$. However other two parameters were well estimated. Close look at the Eqn. (2.4) reveals that $C_{L\alpha}$ and $C_{m\alpha}$ were highly correlated as far as estimation is concerned. Hence, it was decided to fix one of these parameters. Thus $C_{L\alpha}$ was kept fixed to its true value during estimation. Table 3.5 presents the estimated results, which are exactly same as true values used for generation of flight data.

Table 3.5 Comparison of estimated and true value of parameters; scheme 1, flight data S4.

SL. No.	Parameter	True Value	Initial Guess Value	Estimated Value
1.	C_d	0.30	0.5	0.30 (0)*
2.	$C_{L\alpha}$ (Fixed)	2.0	2.0	2.0
3.	$C_{m\alpha}$	3.0	4.0	3.0 (0)
4.	C_{lp}	-0.03	-0.05	-0.03 (0)

*Cramer-Rao bound

After validating the code by above approach, equivalent values of the parameters were estimated using scheme on S5. For this purpose, flight data were generated for a set of different elevations and average values of the parameters, pertaining to each elevation, were estimated. In the estimation process $C_{L\alpha}$ was kept fixed to 2.0. Table 3.6 lists the average values of the parameters estimated for different elevations. Table 3.7 compares measured and estimated range, drift. Referring Table 3.6 it can be observed that roll damping parameter (C_{lp}) exhibit large scatter. This is expected as C_{lp} for shell is a weak derivative (parameter) and its scatter does not effect strongly on range or drift. Notwithstanding this scatter, the comparison of estimated and measured flight data shows excellent matching for all practical purposes.

Table 3.6 Equivalent values of parameters estimated; scheme 1, flight data S5.

SL. No.	Parameter	$\theta = 10^0$	$\theta = 30^0$	$\theta = 45^0$
1.	C_d	0.26810 (0.0000026)	0.27415 (0.0000006)	0.26954 (0.0000003)*
2.	$C_{L\alpha}$ (Fixed)	2.0	2.0	2.0
3.	$C_{m\alpha}$	1.90432 (0.0068)	1.45614 (0.00094)	1.49519 (0.0006)
4.	C_{lp}	-0.07802 (0.00086)	-0.15129 (0.0001)	-0.15915 (0.00006)

*Cramer-Rao bound.

Table 3.7 Comparison of measured and estimated range, drift using equivalent parameters; scheme 1, flight data S5.

SL. No.	Elevation, θ	Range, m		Drift, m	
		Measured	Estimated	Measured	Estimated
1.	10^0	12088.44	12092.48	89.51	91.12
2.	30^0	20390.55	20376.29	386.46	383.873
3.	45^0	23002.13	22866.49	690.06	677.96

3.2.4 Estimation of equivalent values of $C_d, C_{L\alpha}, C_{m\alpha}, C_{y\beta}, C_{n\beta}$ and C_{lp} from S7

In order to test the code flight data were generated as stated in S6. ML method was applied to estimate these parameters. It had been observed that all the four

parameters were estimated together and there values were exactly equal to their true values as shown in Table 3.8. There had been no correlation problem observed here. Due to symmetry of the shell following modeling was incorporated in estimation algorithm i.e. $C_{y\beta} = -C_{L\alpha}$ and $C_{n\beta} = -C_{m\alpha}$.

Strangely enough, the estimation of C_{mq} posed a serious problem. During estimation iteration, the value of C_{mq} was changing sign abruptly from –ve to +ve. At this point the trajectory computation failed to yield any reasonable result. For shell it was found that C_{mq} did not effect range or drift appreciably. Hence it was decided to omit C_{mq} from estimation algorithm.

Table 3.8 Comparison of estimated and true value of parameters; scheme 1, flight data S6.

SL. No.	Parameter	True Value	Initial Guess Value	Estimated Value
1.	C_d	0.30	0.5	0.30 (0)*
2.	$C_{L\alpha}$	2.0	3.0	2.0 (0)
3.	$C_{m\alpha}$	3.0	4.0	3.0 (0)
4.	C_{lp}	-0.03	-0.05	-0.03 (0)

*Cramer-Rao bound

After validating the code by above approach, equivalent values of the parameters were estimated using scheme on S7. However, this scheme failed to

estimate equivalent values of the parameters to any reasonable accuracy when full trajectory data were used. When small part of the flight data (initial 2 sec) was used equivalent values of all the parameters were reasonably estimated. Further analysis is required to investigate the failure of estimation when applied for full trajectory.

3.2.5 Estimation of constant C_d from noisy flight data

In order to investigate the robustness of the present scheme in handling noisy data, S1 was generated with constant C_d and was corrupted with known noise. Using this noisy data once again constant C_d was estimated. Table 3.9 shows the comparison of constant C_d estimated for different noise levels. Observing the values of C_d estimated along with Cramer-Rao bound, it could be concluded that proposed scheme is robust enough to process noisy data to an acceptable level of confidence.

Table 3.9 Comparison of estimated and true values of C_d; scheme 1.

SL.No.	True Value	C_d Estimated with different noise levels			
		5 m	10 m	50 m	100 m
1.	0.30	0.3000004 (0.000002)*	0.3000008 (0.000005)	0.3000043 (0.000026)	0.3000087 (0.000053)

*Cramer-Rao bound

3.3 Estimation of Mach number dependence of aerodynamic parameters using scheme 2

In this scheme in order to capture the Mach number dependence of aerodynamic parameters the modeling in estimation algorithm was done using series approximation of the parameters as a function of Mach number and the constants of this series were

estimated applying ML method.

3.3.1 Estimation of C_d vs Mach number from S1.

In this case flight data S1 were used to estimate C_d varying with Mach number. In this scheme, in estimation algorithm C_d was expressed as a polynomial function of Mach number as $C_d = a0 + a1 * M + a2 * M^2$. The constants $a0$, $a1$ and $a2$ have been estimated through ML method. In order to capture the whole range of Mach number, flight data were generated at different initial velocities of 818m/s and 360m/s at an elevation of 7 Deg. . From the flight data it was observed that the shell when fired at 818m/s covers Mach number range from 2.404 to 1.159. Further when fired at 360m/s, the shell covered Mach number from 1.058 to 0.904. Thus entire range of Mach number was covered.

The values of $a0$, $a1$ and $a2$ estimated for both the charges are presented in Table 3.10. The values of estimated C_d Vs M is given in Table 3.11 for both the charges and is compared with true C_d Vs M in Fig 3.5.

Table 3.10 Estimated values of constants; scheme 2, flight data S1.

SL. No.	V = 818 m/s		V =360 m/s	
	Parameter	Value	Parameter	Value
1.	$a0$	-0.563540 (0.00229)	$a0$	-3.326722 (0.67713)*
2.	$a1$	-0.235335 (0.00257)	$a1$	5.320928 (1.38287)
3.	$a2$	0.041198 (0.00069)	$a2$	-1.694348 (0.70489)

*Cramer-Rao bound

Table 3.11 Estimated C_d Vs M; scheme 2, flight data S1.

SL. No.	Charge (V) = 818 m/s		Charge (V) =360 m/s	
	M	C_d	M	C_d
1.	2.359184	0.237639	1.035000	0.365411
2.	2.309184	0.239790	1.030000	0.356300
3.	2.259184	0.242146	1.025000	0.347105
4.	2.209184	0.244708	1.020000	0.337825
5.	2.159184	0.247477	1.015000	0.328460
6.	2.109184	0.250451	1.010000	0.319011
7.	2.059184	0.253631	1.005000	0.309477
8.	2.009184	0.257018	1.000000	0.299858
9.	1.959184	0.260610	0.995000	0.290154
10.	1.909184	0.264408	0.990000	0.280366
11.	1.859184	0.268413	0.985000	0.270493
12.	1.809184	0.272623	0.980000	0.260536
13.	1.759184	0.277039	0.975000	0.250493
14.	1.709184	0.281661	0.970000	0.240366
15.	1.659184	0.286490	0.965000	0.230154
16.	1.609184	0.291524	0.960000	0.219858
17.	1.559184	0.296764	0.955000	0.209477
18.	1.509184	0.302210	0.950000	0.199011
19.	1.459184	0.307862	0.945000	0.188460
20.	1.409184	0.313721	0.940000	0.177824
21.	1.359184	0.319785	0.935000	0.167104
22.	1.309184	0.326055	0.930000	0.156299
23.	1.259184	0.332531	0.925000	0.145410
24.	1.209184	0.339213	0.920000	0.134436
25.	1.159184	0.346101		

3.3.2 Estimation of C_d vs Mach number from S7

In order to study the effect of sensitivity of trajectory modeling in estimation algorithm, the estimation algorithm only had PMM, whereas the flight data were generated was using complete SDF. It was observed that as far as C_d vs M variation is concerned, simple PMM is adequate enough to capture this variation.

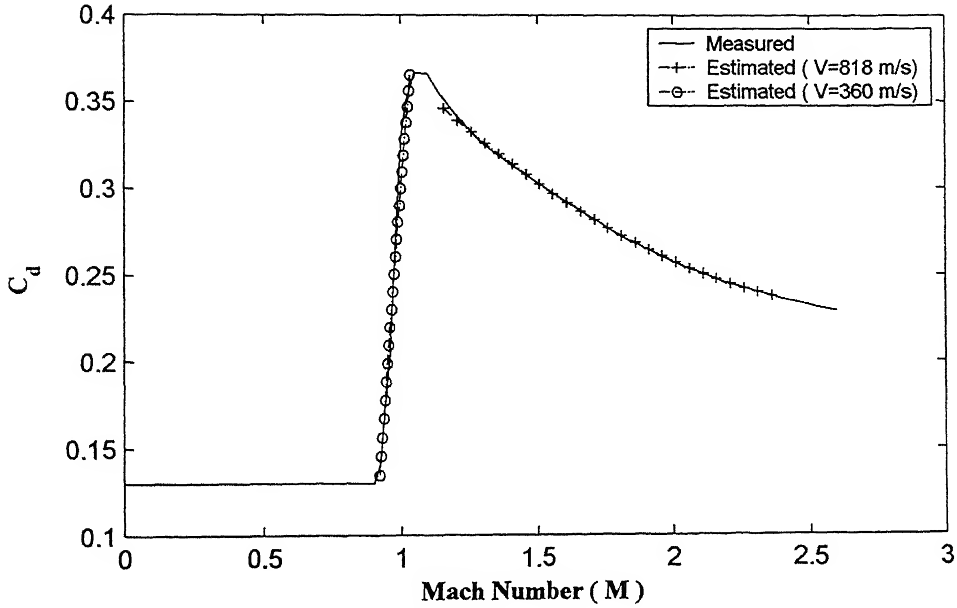

Fig 3.5 Comparison of C_d Vs M between estimated and measured values; scheme 2, flight data S1.

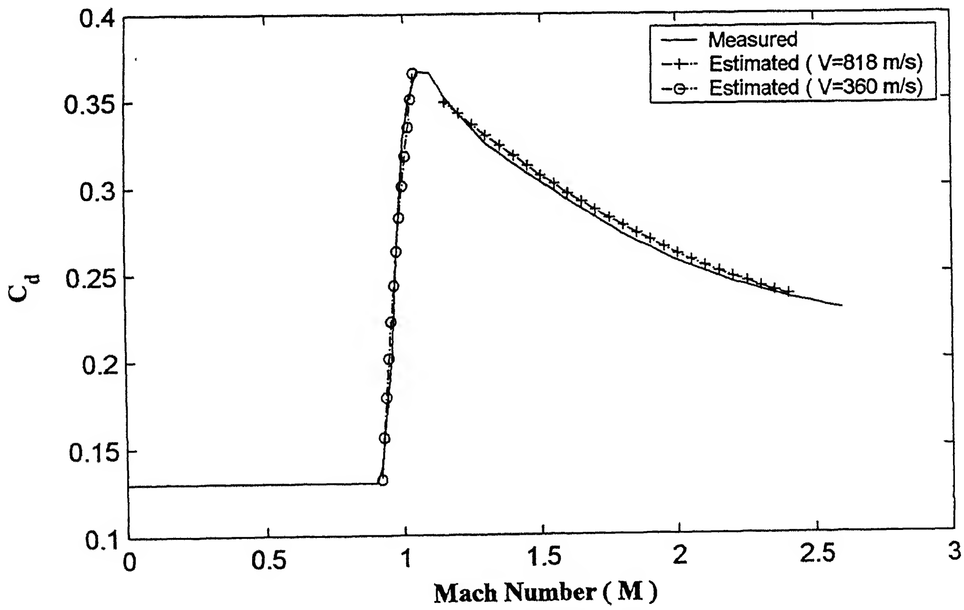

Fig 3.6 Comparison of C_d Vs M between estimated and measured values; scheme 2, flight data S7.

The values of $a0, a1$ and $a2$ estimated for both the charges are given in Table3.12. The values of estimated C_d Vs M is given in Table 3.13 for both the charges and is plotted with true C_d Vs M in Fig 3.6.

Table 3.12 Estimated values of constants; scheme 2, flight data S7.

SL. No.	V = 818 m/s		V =360 m/s	
	Parameter	Value	Parameter	Value
1.	$a0$	-0.548640 (0.00228)	$a0$	-5.449019 (0.6611)*
2.	$a1$	-0.212708 (0.00256)	$a1$	9.736826 (1.3507)
3.	$a2$	0.034592 (0.000690)	$a2$	-3.982691 (0.6887)

*Cramer-Rao bound

3.3.3 Estimation of $C_d, C_{L\alpha}, C_{m\alpha}$ and C_{lp} vs Mach number

Application of scheme 2 to capture Mach number dependence of $C_{L\alpha}, C_{m\alpha}, C_{lp}$ together with C_d posed numerical difficulties. It is recommended that these parameters needed to be expressed using appropriate series expansion. Simple series approximation as was used in case of estimating C_d vs M was not suitable here.

3.4 Estimation of Mach number dependence of aerodynamic parameters using scheme 3

In this scheme in order to capture the Mach number dependence of aerodynamic parameters the estimation was done by splitting the whole trajectory into different sets of points and average values of the parameters pertaining to each set of points were

estimated. The estimated parameters correspond to average Mach number of the set of points considered for estimation.

Table 3.13 Estimated C_d Vs M; scheme 2, flight data S7.

SL. No.	Charge (V) = 818 m/s		Charge (V) =360 m/s	
	M		M	C_d
1.	2.354109	0.239605	1.037632	0.36614
2.	2.304109	0.242184	1.027632	0.351025
3.	2.254109	0.244935	1.017632	0.335113
4.	2.204109	0.24786	1.007632	0.318405
5.	2.154109	0.250957	0.997632	0.3009
6.	2.104109	0.254227	0.987632	0.282599
7.	2.054109	0.257671	0.977632	0.263501
8.	2.004109	0.261287	0.967632	0.243606
9.	1.954109	0.265076	0.957632	0.222915
10.	1.904109	0.269039	0.947632	0.201428
11.	1.854109	0.273174	0.937632	0.179144
12.	1.804109	0.277482	0.927632	0.156063
13.	1.754109	0.281963	0.917632	0.132186
14.	1.704109	0.286617		
15.	1.654109	0.291444		
16.	1.604109	0.296444		
17.	1.554109	0.301617		
18.	1.504109	0.306963		
19.	1.454109	0.312482		
20.	1.404109	0.318174		
21.	1.354109	0.324038		
22.	1.304109	0.330076		
23.	1.254109	0.336287		
24.	1.204109	0.342671		
25.	1.154109	0.349227		

3.4.1 Estimation of C_d Vs Mach number

In this case the flight data S1 was used to estimate C_d varying with Mach number. In this scheme the whole trajectory (flight data) was split into different sets of

100 points each. Then an average value of C_d, pertaining to each set of points, was estimated by processing these sets of points. The initial conditions (velocity and elevation (θ)) required for generation of estimated response for each set of points was calculated by differentiating x and y with respect to time. The estimated C_d Vs M is plotted against original C_d Vs M as shown in Fig 3.7 for different elevations and the respective values, for an elevation of 10 Deg., are shown in Table 3.14. It can be seen from Table 3.14 and figures that the estimated values matched closely with that of the true values.

3.4.2 Estimation of $C_d, C_{L\alpha}, C_{m\alpha}$ and C_{lp} vs Mach number

In order to estimate the functional dependence of all the parameters with Mach number we require all the initial conditions of the motion variables. As the estimation algorithm was made exhaustive, using MPMM and SDF models, the number of initial conditions required became large. It was not possible to get all these initial conditions through spatial coordinates (x, y, z) only. This was only possible when estimation algorithm used PMM.

3.4.3 Estimation Of C_d Vs Mach number from noisy data

In this case the flight data generated in S3 (noisy data) was used to estimate C_d varying with Mach number. To apply this scheme, for estimating C_d Vs M, differentiation of x and y were essential. For noisy data differentiation could not be done to yield reasonable result as presented in Fig 3.8. To overcome this difficulty, the noisy data was smoothened by fitting a polynomial.

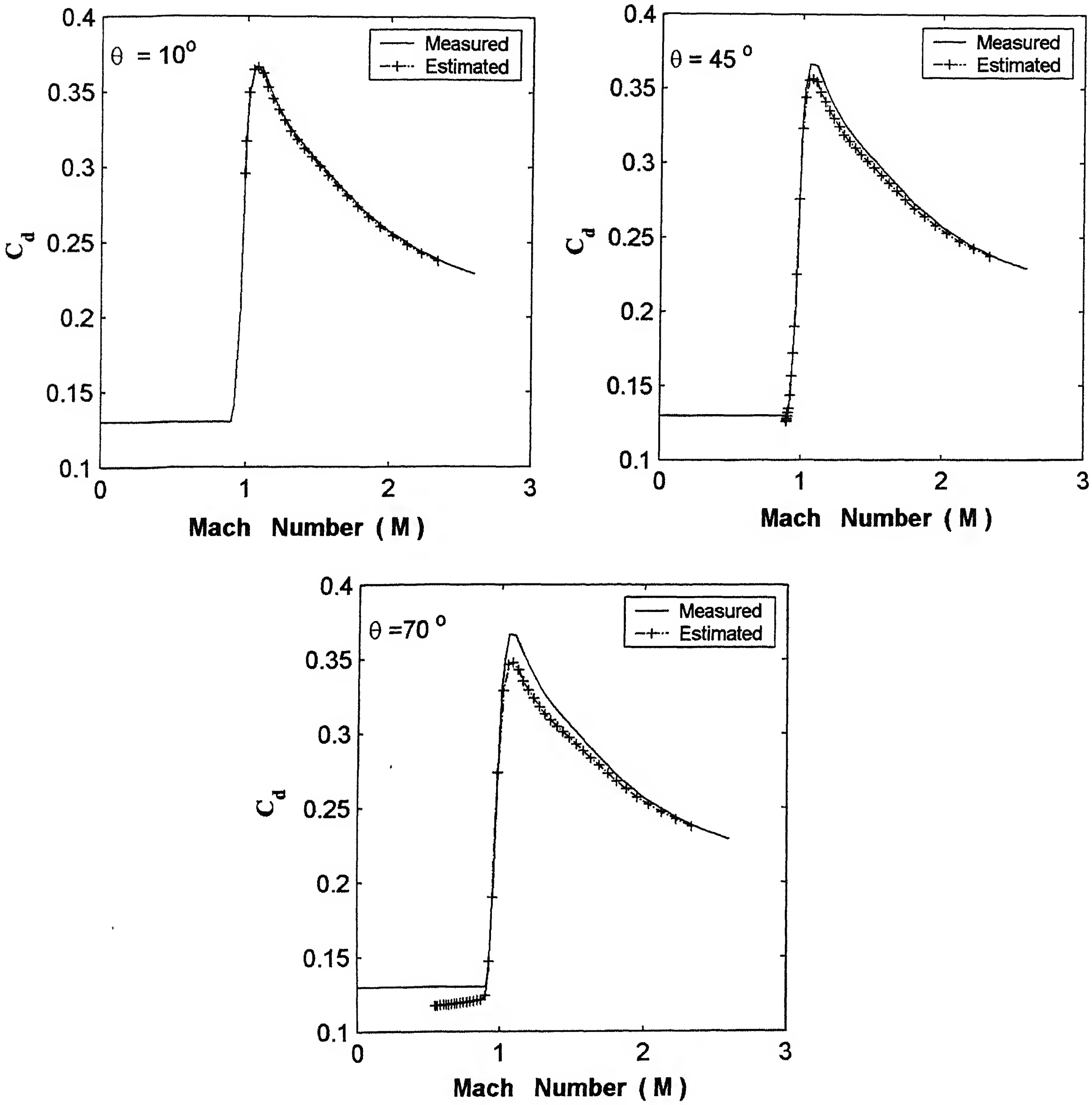

Fig. 3.7 Comparison of C_d vs M between estimated and measured values; scheme 3, flight data S1.

Table 3.14 Estimated C_d Vs M; scheme 3, flight data S7.

SL. No.	Mach Number	C_d
1.	2.340629	0.237474 (0.002630)*
2.	2.226243	0.242646 (0.002950)
3.	2.121262	0.248222 (0.003291)
4.	1.934627	0.260397 (0.004040)
5.	1.851056	0.267130 (0.004449)
6.	1.773072	0.273649 (0.004882)
7.	1.700093	0.280969 (0.005339)
8.	1.631685	0.287871 (0.005820)
9.	1.567552	0.294438 (0.006323)
10.	1.450988	0.306815 (0.007394)
11.	1.398073	0.312466 (0.007959)
12.	1.301720	0.324071 (0.009138)
13.	1.257749	0.331029 (0.009751)
14.	1.216238	0.338202 (0.010378)
15.	1.177040	0.345416 (0.011018)
16.	1.104875	0.362316 (0.012331)
17.	1.071931	0.366093 (0.012994)
18.	1.041634	0.364824 (0.013639)
19.	1.014675	0.349604 (0.014232)
20.	0.992291	0.317082 (0.014728)

* Cramer-Rao bound.

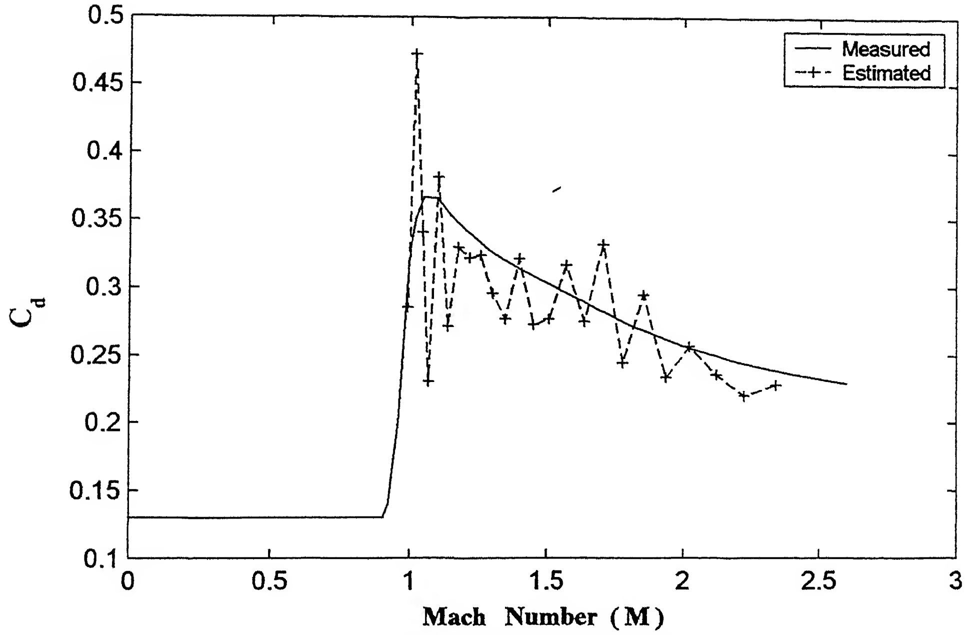

Fig 3.8 Comparison of C_d Vs M between estimated and measured values; scheme 3, flight data S3.

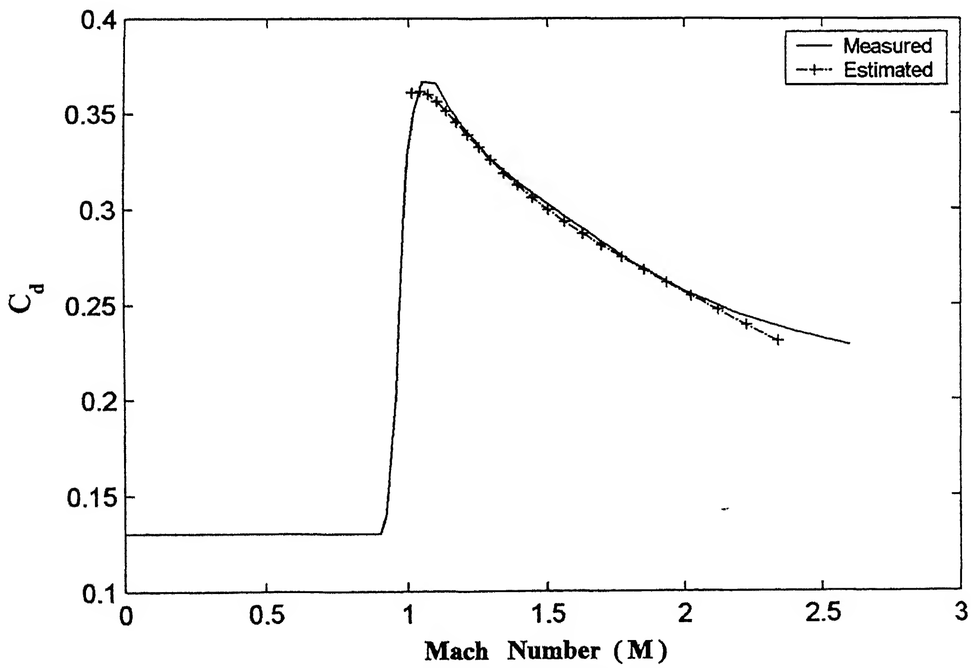

Fig 3.9 Comparison of C_d Vs M between estimated and measured values after smoothening; scheme 3, flight data S3.

For an elevation of 10 Deg. the noisy data is approximated using a 6^{th} order polynomial given by:

$$\begin{aligned}\text{Range: } & -3.6\text{e-}07*x^6+0.00013*x^5-0.011*x^4+0.513*x^3 \\ & -20.05*x^2+804.12*x+0.718.\end{aligned} \tag{3.1}$$

$$\begin{aligned}\text{Height: } & -1.47\text{e-}06*y^6+0.00013*y^5-0.0058*y^4+0.193*y^3 \\ & -8.57*y^2+142.15*y-0.103.\end{aligned} \tag{3.2}$$

The flight data, smoothened using the polynomial approximation, was used for estimation. The estimated C_d Vs Mach number compares well with that of true C_d Vs M as shown in Fig 3.9. For future work, flight data may be processed using standard filtration techniques before being used for parameter estimation.

3.5 Estimation of Mach number dependence of aerodynamic parameters using scheme 4

3.5.1 Estimation of C_d, $C_{L\alpha}$, $C_{m\alpha}$ and C_{lp} vs Mach number

In order to estimate the dependence of all the parameters at a time with Mach number, the earlier schemes were inadequate. In order to avoid such difficulties an alternate scheme have been proposed for field application.

Flight data were generated corresponding to different charges (velocities) and the initial part of the trajectory was used for estimation. In field application this amounts to firing few rounds of different charges at some elevation. Initial 200 points of the trajectory were taken to estimate average values of the parameters (C_d, $C_{L\alpha}$, $C_{m\alpha}$, C_{lp}) for different charges respectively. Hence the equivalent parameters

estimated belong to the average Mach number for the set of 200 points. Thus for different charges fired we get equivalent parameters corresponding to that Mach number. The aerodynamic parameters thus estimated as function of Mach number are presented in Fig 3.10. Referring Fig 3.10, it can be observed that for most of the parameters, the Mach number dependence was well estimated. However estimated value of C_{l_p} showed large scatter.

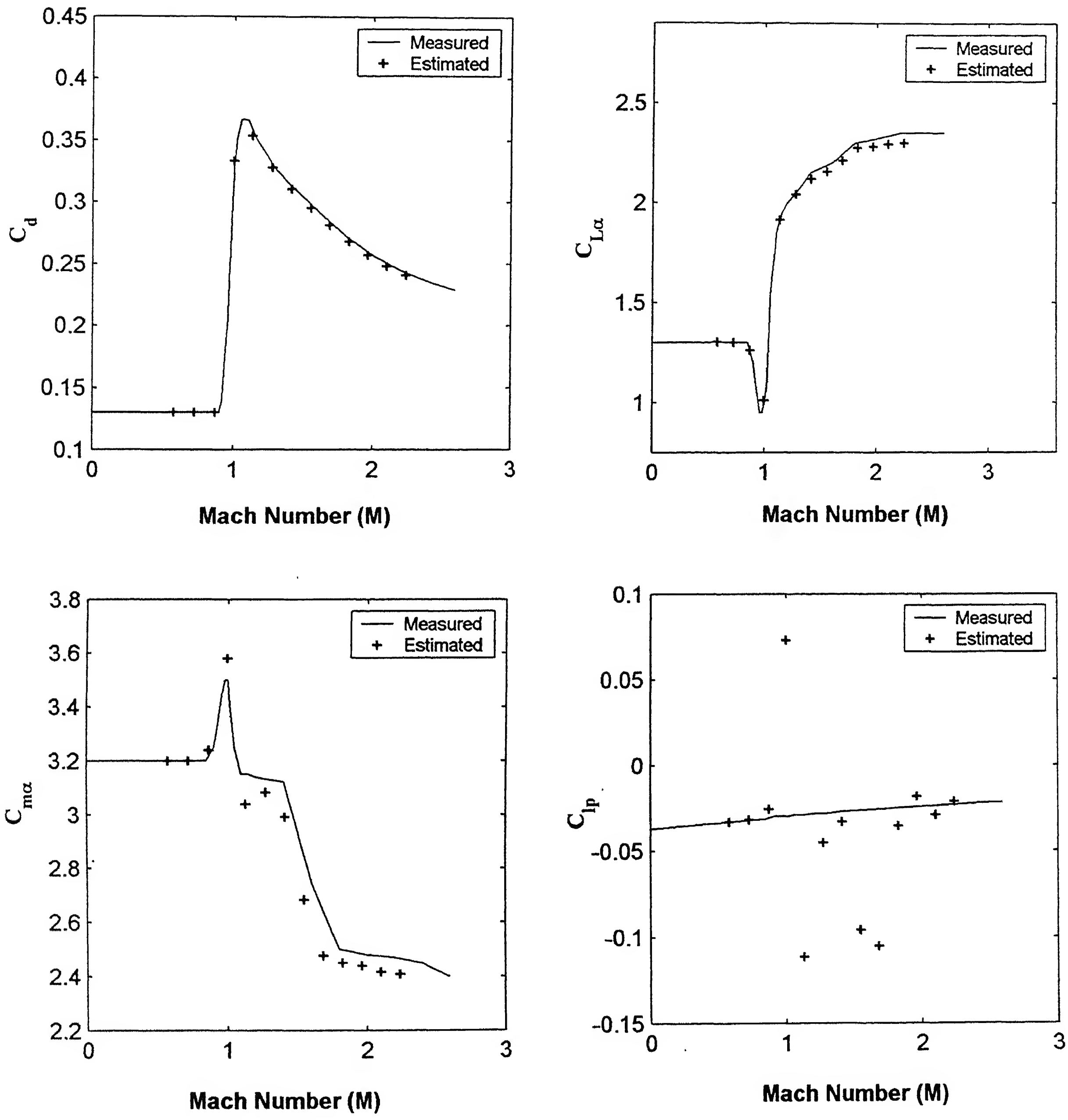

Fig 3.10 Comparison between measured and estimated parameters; scheme 4, flight data S7.

CHAPTER 4

CONCLUSIONS AND FUTURE SUGGESTIONS

4.1 Conclusions

In the present work, schemes were proposed to estimate both equivalent and Mach number dependence of aerodynamic parameters from radar tracked flight data (x, y, z) of an artillery shell. Four schemes using ML method are proposed for parameter estimation. The proposed schemes show promise to estimate the aerodynamic parameters.

Scheme 1 is used to estimate equivalent aerodynamic parameters and scheme 2 to 4 are used to estimate functional dependence of parameters on Mach number. Scheme 2 assumes series approximation of the parameters as a function of Mach number in estimation algorithm. In scheme 3, whole trajectory was split into different sets of points and average values corresponding to each set of points were estimated. In scheme 4 the flight data was generated at a particular elevation corresponding to different charges and small initial part of the trajectory was used to estimate the estimate the Mach number dependence of parameters.

Based on all the schemes it can be concluded that scheme 1 can be suitably applied to get average values of aerodynamic parameters. The differences in range/drift using the average values are well within 1% of the maximum range/drift. Scheme 2 and 3 are best suited for capturing the Mach number dependence of C_d. However scheme 3 faced difficulties in order to estimate the Mach number dependence of all the parameters, scheme 4 was best suited for this.

4.2 Suggestions for future work

1. In the present work, simulated flight data were used to estimate aerodynamic parameters of the artillery shell due to non-availability of the real flight data. In order to enhance the capability of the proposed schemes, these should be validated using real flight data.

2.To deal with noisy data better filtration techniques should be used to pre process the data before used for parameter estimation.

3. Scope of the scheme 2 may be expanded to capture Mach number dependence of all the parameters by assuming appropriate model, representing the variation of parameters with Mach number, in estimation algorithm.

4. All the schemes were applied to the simulated data generated for standard atmospheric conditions. Applicability of the proposed schemes to estimate parameters using flight data under non-standard atmospheric conditions may be explored.

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APPENDIX A

MAXIMUM LIKELIHOOD (ML) METHOD

The parameter estimation algorithm used in this study is the conventional 'Maximum Likelihood Method'. The programs were written in 'C' using the MLE algorithm. The algorithm is given as follows:

Let a mathematical model of a system be represented by the following equations:

$$X(t_i) = A\,X(t_i) + B\,U(t_i) \qquad \text{(A.1)}$$

$$Z(t_i) = C\,X(t_i) + G\,n_t \qquad \text{(A.2)}$$

where X is the state vector, U the control vector, Z the measured (observation) response and n_i is the noise vector respectively. The matrices A and B contain the unknown stability and control derivatives (parameters). If there is no state noise and the matrix GG is known, then the Maximum Likelihood estimator minimizes the cost function,

$$J(\theta) = (1/2)\sum_{i=1}^{n}\left\{\left[\,Z(t_i) - Z_\theta(t_i)\,\right]^T \left(GG^T\right)^{-1}\left[\,Z(t_i) - Z_\theta(t_i)\,\right]\right\} \qquad \text{(A.3)}$$

where GG^T is the measurement noise covariance matrix and $Z_\theta(t_i)$ is the computed response estimate of Z at t_i for a given value of the unknown parameter vector θ. The cost function is a function of the difference between the measured and computed time histories. In our case, we have taken $GG^T = 1$ for the cost function. To minimize the cost

function $J(\theta)$, we can apply the Newton-Raphson algorithm, which chooses successive estimates of the vector θ of unknown coefficients. If k is the iteration number, then

$$\theta_{k+1} = \theta_k - \left[\nabla_\theta^2 J(\theta_k) \right]^{-1} \left[\nabla_\theta^T J(\theta_k) \right], \tag{A.4}$$

where the first gradient is defined by

$$\nabla_\theta J(\theta) = -\sum_{i=1}^{n} \left\{ \left[Z(t_i) - Z_\theta(t_i) \right]^T \left(GG^T\right)^{-1} \left[\nabla_\theta Z_\theta(t_i) \right] \right\}, \tag{A.5}$$

and the second gradient is approximated by

$$\nabla_\theta^2 J(\theta) = \sum_{i=1}^{n} \left\{ \left[\nabla_\theta Z_\theta(t_i) \right]^T \left(GG^T\right)^{-1} \left[\nabla_\theta Z_\theta(t_i) \right] \right\}. \tag{A.6}$$

The matrix GG^T is approximated by a diagonal matrix, with the diagonal elements given by,

$$\sum_{i=1}^{n} \frac{\left\{ \left[Z(t_i) - Z_\theta(t_i) \right] \left[Z(t_i) - Z_\theta(t_i) \right]^T \right\}}{N}, \tag{A.7}$$

where N is total number of data points. The term $\nabla_\theta Z_\theta(t_i)$ occurring in the first and the second gradients is evaluated by finite difference approximation for each of the sensitivity coefficients present in it. For example, to evaluate sensitivity coefficient $\dfrac{\partial \alpha}{\partial C_{mq}}$, the equations of motion are solved to obtain α for $C_{mq} + \Delta C_{mq}$ and $C_{mq} - \Delta C_{mq}$.

Let the respective values of α be α^+ and α^-, then we have

$$\frac{\partial \alpha}{\partial C_{mq}} = \frac{\alpha^+ - \alpha^-}{2\Delta C_{mq}}$$

The reliability of the estimated parameter in terms of the Cramer-Rao bounds is given by the square root of the diagonal elements of $\left[\nabla_\theta^2 J(\theta_k)\right]^{-1}$, which is anyway evaluated as part of the ML algorithm.

In the present work, for estimating the aerodynamic parameters of the artillery shell, the vector of unknown parameters is given by $\theta = \left[\ C_d, C_{L\alpha}, C_{m\alpha}, C_{lp}\ \right]$ for a typical case with the $Z(t_i)$ and $Z_\theta(t_i)$ responses being computed by the spatial coordinates x, y and z respectively.